Algebraic geometry 2

Exercise Sheet 11

PD Dr. Maksim Zhykhovich Summer Semester 2026, 30.06.2026

Exercise 1. Show that the scheme $X \times_S Y$ from Step 2 in the proof of Theorem 4.31, which we constructed by glueing $U_i \times_S Y$, satisfies the universal property of the fibred product of X and Y over S.

Exercise 2. Let $f : X \to Y$ be a morphism of schemes and let $y \in Y$. Denote by $X_y := X \times_Y k(y)$ the fiber of f above y. Show that the underlying topological space of the scheme X_y is homeomorphic to the set-theoretical pre-image $f^{-1}(\{y\}) \subset X$ (with the subspace topology).
Hint: Reduce to the case when X and Y are affine.

Exercise 3. Let k be an algebraically closed field of characteristic not 2.
Let $P := \operatorname{Spec} k[X,Y]/(Y - X^2)$ and $\mathbb{A}^1_k := \operatorname{Spec} k[Y]$. Let $f : P \to \mathbb{A}^1_k$ be a morphism of schemes corresponding to the ring homomorphism

$$k[Y] \to k[X,Y]/(Y - X^2), Y \mapsto \text{class of } Y.$$

Describe the fiber P_b for every point $b \in \mathbb{A}^1_k$.